On a Simple Method of Accurate Surveying with an Ordinary Camera. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Measurements of photographs have proved so accurate and convenient in astronomy that they will probably be found useful elsewhere. The present paper suggests an extension of the methods adopted for treatment of the Astrographic Chart plates to terrestrial surveying; and although not definitely astronomical, since it arises out of astronomical work, it may perhaps not unsuitably be communicated to this Society.

2. *Uses of the camera in surveying hitherto.*—Instruments designed for using the camera in terrestrial surveys have hitherto started with the theodolite as a substructure. Now the theodolite is primarily designed to measure *angles*, and I hope to show in the present paper that the use of angles is unnecessary, and only complicates the formulæ. It is not an angle itself but the tangent of an angle which arises fundamentally in the work, when a view of any kind is projected on a plane; and if we keep to plane geometry with straight lines and linear formulæ, we get much simpler formulæ; just as in astronomical work it simplifies matters to use rectilinear coordinates instead of curvilinear or angular. This has not been realised (so far as I have been able to find out) in surveying any more than it was in astronomy; indeed existing instruments for photographic surveying often contain a "scale of tangents" designed to enable the surveyor to convert the convenient linear measures with which the photograph directly furnishes him into angular measures, which are really more inconvenient but to which he has become accustomed from use of the theodolite.

When the photographs have been obtained methods of making use of them have usually been graphical; and the positions of points have been laid down by the "method of intersections." Now graphical methods are limited by practical considerations, such as size of paper; and though in many cases they may be all that is necessary, it is convenient to have a more complete method. In the present paper it is shown how accurate measurements of the plate can be used, without any troublesome calculations, to give all the required data. Thus we may state the

3. *Advantages of the present method* under two heads: (*a*) the use of linear measures and formulæ entirely, instead of angles, and (*b*) the substitution of accurate measurement of the plate for graphical methods of treating the photographs. Under head (*a*) is to be ranged the considerable practical gain of doing away with any complicated or expensive mounting, and using a camera of the ordinary pattern.

4. *Principle of the method.*—Let A be the centre of a lens forming an image on a plate GE, and let the image of a point

P fall at G. We shall suppose for the present that there is no optical distortion, *i.e.* that the images on the plate all fall truly on rays through the point A; and the way of dealing with any deviation from this law will be considered later.

Let AE be drawn normal to the plate from the centre A. The point E, which is the foot of the normal from the lens centre on the plate, is of some importance and will be called the plate centre. Let AE $=f$. It is the focal length of the lens when an infinitely distant object is in focus, and in all cases is of comparable dimensions with this focal length.

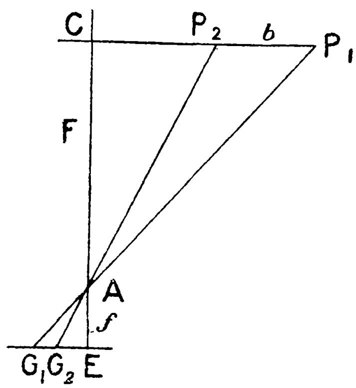

Fig. 1.

The coordinates of P in space referred to axes AC, CP are

$$AC=F \text{ say, and } CP=X$$

and if we put EG, the measured distance of the image from the plate centre, equal to x, we have of course

$$x/f=X/F \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

From measurements on a single photograph we can get nothing more than this—the ratio X/F. To determine either of these completely we must take another photograph. Now it will clearly be of advantage to keep one or other of these quantities the same in both pictures. If we keep F the same we must move along a line parallel to the plate; if X the same, along a line perpendicular to the plate. Both these plans have special advantages, and we will consider them each in turn.

5. *The parallactic or stereoscopic method.*—Let us move the camera a measured distance b to the right, keeping it parallel to itself. (The practical method of securing these conditions will be considered presently.) Then this is equivalent to supposing all the objects in the field of view moved a distance b to the left

as in fig. 1. Denote the first and second positions by the suffixes 1 and 2; so that we have

$$\left.\begin{aligned} x_1/f&=X_1/F \\ x_2/f&=X_2/F \\ \text{and} \qquad X_1-X_2&=b \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

whence $(x_1-x_2)/f=b/F$ ... ... (3)

or $$F=\frac{bf}{x_1-x_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Now bf is a constant for the pair of pictures. Thus to get the distance F of any object measured in a direction perpendicular to the plate, or the base, we measure the distances x_1 x_2 on the two plates, take their difference, and divide it into a constant.

Again $$X_1=\frac{F}{f}\,.\,x_1=\frac{b\,x_1}{x_1-x_2} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

so that the other coordinate (on the ground plan) of the object is obtained equally simply. Finally if we want the elevation of the object, call its height above the horizontal plane AGE through the lens centre Y, and let y be the measured height of the image above the line EG on either picture. Then clearly

$$Y=\frac{F}{f}\,.\,y=\frac{b\,y}{x_1-x_2} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Thus the actual coordinates XYF of the object in space are obtained from the measures on the plate x, y, and the focal length f on multiplying by the factor $b/(x_1-x_2)$. But since this may also be written in the form F/f and is constant, it follows (as might have been expected) that we can increase x_1-x_2 to any desired extent by increasing b.

6. *Convenient size of base.*—A few figures may be given at once to show the kind of base that will give good results. Suppose f (the focal length of the camera) to be 1 foot, and let b be 100 feet. This is a comparatively modest base, which would generally be available by selecting positions with care. Then if x_1-x_2 be a quarter of an inch, or 0·021 feet,

F=100/0·021=4,800 feet, or nearly 1 mile.

Now, how accurately can we measure the difference x_1-x_2 and thus determine the distance of objects a mile away? Experience has shown that it is easy to measure a photograph within, say, a thousandth of an inch—no one will think this too severe a limit. It corresponds to 1″·5 on the astrographic plates, and most observers would concede a division by at least 10. But we must remember that terrestrial objects may not be as well defined as stars, and so we will take an outside limit, say 0·001 inch. This

would correspond to an error of 1/250 of the distance at a mile away, say 20 feet; and this is considerable accuracy. For other distances the error is as the square of the distance, *e.g.* at 5 miles it would be 500 feet.

7. *Second method. Advance in direction of view.*—The formulæ, when we keep X the same, are equally simple. We have

$$\left.\begin{aligned} f/x_1 &= F_1/X \\ f/x_2 &= F_2/X \\ F_1 - F_2 &= b, \text{ say} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus
$$f\left(\frac{1}{x_1}-\frac{1}{x_2}\right)=b/X$$

or
$$\left.\begin{aligned} X &= \frac{b\,x_1x_2}{f(x_2-x_1)} \\ F_1 &= \frac{bx_2}{x_2-x_1} \\ Y_1 &= \frac{F_1}{f}y_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$
and obviously

The weak point in this case is that when $x_1=0$ we have $x_2-x_1=0$: that is, the method fails to give results for objects near the middle of the picture. There is, however, a corresponding advantage in the adjustments of the instrument, as will presently appear.

8. *Necessary conditions.*—It remains to consider how to secure the necessary conditions, which are:

(1) To measure a base of a given length in a given direction. The practical difficulties here are such as need not concern us.

(2) To keep the camera parallel to itself. For this the camera should first be fitted with at least two cross levels. No reversal of the levels is necessary, as it is only necessary that the camera should be parallel to itself; but if the levels have serious errors, the survey will not be referred to a true horizontal plane. The levels will check the parallelism so far as rotations about horizontal axes are concerned. As regards that about a vertical axis,

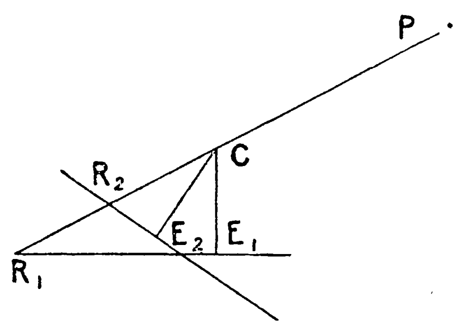

Fig. 2.

remark that in the second method this adjustment is checked by the pictures themselves, for objects at the middle of the picture

should not be displaced at all; and any displacement at once shows lack of parallelism. There is no difficulty in applying the correction for lack of parallelism, which takes a form already familiar in astronomical work. Let C be the centre of the lens, E_1R_1, E_2R_2, the correct and displaced positions of the plate; P an object which forms images at R_1, R_2.

Then
$$\left.\begin{aligned} E_1R_1 &= f \tan\theta \\ E_2R_2 &= f \tan(\theta-\alpha) \end{aligned}\right\} \quad \ldots \ldots \quad (9)$$

where α is the angle of displacement and may be supposed small.

Hence
$$E_2R_2 = f \tan\theta - f\alpha \sec^2\theta$$

or
$$E_2R_2 - E_1R_1 = -f\alpha\left[1+\left(\frac{E_1R_1}{f}\right)^2\right] \quad \ldots \quad (10)$$

Thus the displacement of an object near the middle of the plate is nearly constant and measures the constant $f\alpha$ with accuracy. And for other objects we multiply this correction by a simple factor.

9. For the first or stereoscopic method the determination of this fundamental adjustment is not quite so simple, as all the objects in the picture are shifted. A compass-needle on the camera would not give the parallelism with sufficient accuracy when the base is small. If there is one object in the picture whose distance is accurately known, the error of parallelism could be found; but if the object is near, a considerable accuracy in its distance is necessary. The best general methods of securing this parallelism would probably be found only by experience; but that there is no essential difficulty may be seen from the following suggestions.

(*a*) If a cross telescope were attached to the camera it could be pointed on a distant object, the same in each position; or,

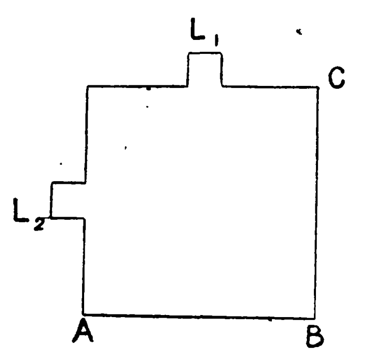

Fig. 3.

better still, two objects in line, such as two staffs set up for the purpose.

(*b*) Suppose a camera fitted with two lenses in two adjacent

faces ; L_1 giving a picture on a plate AB, and L_2 on a plate BC. Let both pictures be taken without moving the camera. Then advance 100 feet, either in the direction BC or BA, and take another pair of pictures. If in the direction BC, then the pair with lens L_1 will give an accurate measure of the error of parallelism, which can be used to reduce the pair of stereoscopic pictures taken with L_2. It should be remarked that fig. 3 is only the ground plan of the cameras, which can be separate instruments firmly attached together with lenses at right angles. The image formed by one lens could be reflected by a diagonal reflector on to the same plate as that formed by the other.

(*c*) If only one camera is available it can be made to serve as a pair at right angles, even without any accurate instrument for measuring angles. Nail two laths with straight edges at right angles to one another on a board, and place some fiducial edge of the camera against each in turn, taking care not to move the board between the exposures. There should be no real difficulty in securing this adjustment even in field work.

10. *Form of camera.*—It will be seen that no special form of camera is necessary, and probably the most convenient will be the simplest : a rectangular box with no focal adjustment, or better, two such firmly attached at right angles, would give good results. Either films or plates might be used, but if films, then a plate of réseau lines should be fixed immediately in front of the film. In any case there should be at least one set of cross-wires to identify the origin for measures on the two plates.

11. *Optical distortion* need not be a trouble even though it be large, as may happen with a wide-angle camera. It should be *determined* thus : expose the camera to any view first with the aperture which is to be used in practice ; secondly with a pinhole stop : measures of the two pictures will at once give the correction for distortion at different distances from the optical centre.

12. *The measures* should be made by the method of rectangular coordinates, in any of the ways found convenient for star measures. A series of réseau lines impressed on the plate is almost a practical necessity, but there is no need to dwell on points of this kind which have already been worked out in the case of star-measures. It may be remarked incidentally that a good deal of useful work might be done without taking photographs at all, by measuring the positions of images on the ground glass of the camera, or a substitute. If the substitute be a photograph of réseau lines, and an eyepiece with a cross-scale attached to it be placed carefully in contact with this photograph, so as to focus objects in the field of view, the réseau lines, and the cross-scale altogether, measures of position could be made at once, and a few such would doubtless be a valuable check on the photographic measures.

13. *General plan of survey.*—The following general plan of operations may thus be suggested. On any route, however

irregular, there will be opportunities for selecting pairs of stations accurately in the line of march, and 100 feet apart (or some other convenient distance). By pairs of photographs taken as above the positions of near objects can be determined well, and those of distant objects not so well. But these latter will appear on many photographs, and we thus get longer bases from which to determine them more accurately. A compass needle attached to the camera, though useless for avoiding minute differences of orientation, would be very useful in comparing results at distant stations.

On the Accuracy of Measures on Photographs: Remarks on recent Papers by M. Loewy and Mr. H. C. Plummer. By Arthur R. Hinks, M.A.

In the supplementary number of the *Monthly Notices*, 1901 (vol. lxi., p. 618), Mr. H. C. Plummer has recently discussed the very elaborate memoir by M. Loewy, "Sur la précision des mesures des coordonnées rectilignes des images stellaires," in the eighth circular of the *Conférence astrophotographique internationale de Juillet* 1900. M. Loewy has sought to find a general formula which shall give the probable error of the mean of a number of settings upon a star image, in a number of different orientations of the plate; and concludes that to avoid systematic error it is necessary to measure the plate in four orientations. Mr. Plummer has rediscussed the figures which M. Loewy gives, and comes to a number of conclusions, of which the most important are:

"(4) By combining measures made in the direct and reversed positions of the plate a much better result is obtained, to which M. Loewy's formula does *not* apply.

"(6) The error eliminated by reversing the plate is definite for each image, but may vary both in magnitude and in sign from one star to another."

Briefly, the principal point in dispute is, Can a sensible compensation of the systematic errors of measurement be obtained by measuring the plate in two positions whose orientations differ by 180°?

My excuse for intruding into the discussion between M. Loewy and Mr. Plummer at this point is as follows: For some months I have had the pleasure of being in correspondence with M. Loewy in reference to certain aspects of the question; he has been so kind as to give me very full and interesting explanations of his views; and I have his permission to give some passages from his letters in the present note.

At the beginning of his memoir (*loc. cit.*) M. Loewy says:

"Il y a donc à distinguer, lorsqu'on détermine une co-